

Modelling and Prediction of the MXNUSD Exchange Rate Using Hybrid RLS-BP Interval Type-1 Non-Singleton Type-2 Fuzzy Logic Systems

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Abstract. This paper presents a novel application of the interval type-1 non-singleton type-2 fuzzy logic system (FLS) to one step ahead prediction of the daily exchange rate between Mexican Peso and US Dollar (MXNUSD) using the recursive least-squared (RLS)-back-propagation (BP) hybrid learning method. Experiments show that the exchange rate is predictable and according to a simple short-term investment strategy, a good annual profit rate can be obtained. A non-singleton type-1 FLS and an interval type-1 non-singleton type-2 FLS, both using only BP learning method, are used as a benchmarking systems to compare the results of the hybrid interval type-1 non-singleton type-2 FLS (RLS-BP) forecaster.

Keywords: Type-2 fuzzy inference systems, type-2 neuro-fuzzy systems, hybrid learning, uncertain rule-based fuzzy logic systems.

1 Introduction

Interval type-2 fuzzy logic systems (IT2 FLS) constitute an emerging technology [1]. As in hot strip mill process [2], [3], in autonomous mobile robots [4], and in plant monitoring and diagnostics [5], [6], financial systems are characterized by high uncertainty, nonlinearity and time varying behavior [7], [8]. This makes it very difficult to forecast financial variables such as exchange rate, closing prices of stock indexes and inflation. The ability of comprehending as well as predicting the movements of economic indices could result in significant profit, stable economic environments and careful financial planning. Neural networks are very popular in financial applications and a lot of work has been done in exchange rate and stock markets predictions described elsewhere [9]–[11].

T2 fuzzy sets let us model the effects of uncertainties, and minimize them by optimizing the parameters of the IT2 FS during a learning process [12]–[14]. Although some econometricians claim that the raw data used to train and test the FLS forecasters should not be directly used in the modelling process, since it is time varying and contains non-stationary noise, they were used by us to directly train the IT2 FLS. The inputs were modeled as type-1 (T1) non-singletons, incorporating the

uncertainties of the training data in the antecedents and consequents of the fuzzy rule base. The interval type-1 non-singleton type-2 fuzzy logic system (IT2 NSFLS-1) forecaster accounts for all of the uncertainties that are present in this application, rule uncertainties due to training with the noisy data and measurement uncertainties.

2 MXNUSD Exchange Rate Prediction

2.1 Noisy Input Output Data Pairs

The data used to train the three forecasters cover a period of nine years and four and a half months from 01/02/97 to 06/17/06 whereas the test data cover five months from 06/18/06 to 10/12/05. The daily closing price of MXNUSD exchange rate was found on the Web site: <http://pacific.commerce.ubc.ca/xr/>. It was a set of $N = 2452$ noisy data, $x(1), x(2), \dots, x(2452)$. It is assumed that the noise free sampled MXNUSD exchange rate, $s(k)$, is corrupted by uniformly distributed stationary additive noise $n(k)$, so that

$$x(k) = s(k) + n(k) \quad k = 1, 2, \dots, 2452 \quad (1)$$

and that signal to noise ratio (SNR) is equal to 0 dB. Figure 1, shows the trend of the raw data. The first 2352 noisy data were used for training, and the remaining 100 data were used for testing the forecasters. Four antecedents were used as inputs, $x(k-3), x(k-2), x(k-1)$ and $x(k)$, to predict the output, $y = x(k+1)$.

2.2 IT2 NSFLS-1 Design

The architecture of the IT2 NSFLS-1 was established in such away that its parameters were continuously optimized. The number of rule-antecedents (inputs) was fixed to four. Each antecedent-input space was divided into three FS, resulting in 81 rules. Gaussian primary membership functions (MF) of uncertain means were chosen for the antecedents. Each rule of the IT2 NSFLS-1 was characterized by 12 antecedent MF parameters (two for left-hand and right-hand bounds of the mean and one for standard deviation, for the FS of each of the four antecedents), and two consequent parameters (one for left-hand and one for right-hand end points of the consequent type-1 FS), giving a total of 14 parameters per rule. Each input value had one standard deviation parameter, giving 4 additional parameters.

The resulting IT2 NSFLS-1 used type-1 (T1) non-singleton fuzzification, join under maximum t-conorm, meet under product t-norm, product implication, and center-of-sets type-reduction.

The IT2 NSFLS-1 was trained using two main learning mechanisms: the back-propagation (BP) method for tuning of both antecedent and consequent parameters, and the hybrid training method using recursive least-squared method (RLS) for tuning

of consequent parameters as well as the BP method for tuning of antecedent parameters. In this work, the former is referred to as IT2 NSFLS-1 (BP), and the latter as hybrid IT2 NSFLS-1 (RLS-BP).

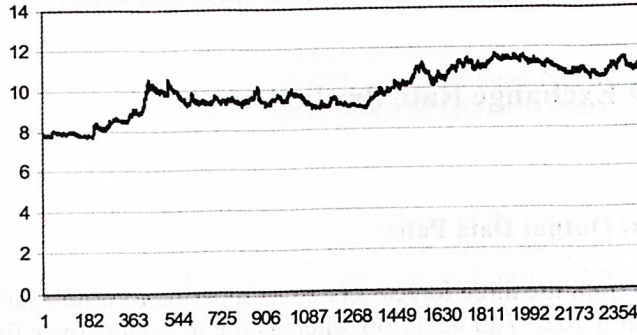


Fig. 1. The daily closing price of MXNUSD exchange rate.

2.3 IT2 Fuzzy Rule Base

The IT2 NSFLS-1 fuzzy rule base consists of a set of IF-THEN rules that represents the model of the system. This IT2 NSFLS-1 has four inputs $x_1 \in X_1$, $x_2 \in X_2$, $x_3 \in X_3$, $x_4 \in X_4$ and one output $y \in Y$. The rule base has $M = 81$ rules of the form

$$R^i : IF \ x_1 \text{ is } \tilde{F}_1^i \text{ and } x_2 \text{ is } \tilde{F}_2^i, \text{ and } x_3 \text{ is } \tilde{F}_3^i \text{ and } x_4 \text{ is } \tilde{F}_4^i, \text{ THEN } y \text{ is } \tilde{G}^i. \quad (2)$$

where $i = 1, 2, 3, \dots, 81$ rules; $\tilde{F}_1^i, \tilde{F}_2^i, \tilde{F}_3^i$ and \tilde{F}_4^i are the antecedent type-2 fuzzy sets, and \tilde{G}^i is the consequent type-2 fuzzy set of rule i .

2.4. Antecedent Membership Functions

The primary MF for each antecedent is an IT2 FS described by Gaussian primary MF with uncertain means:

$$\mu_k^i(x_k) = \exp \left[-\frac{1}{2} \left[\frac{x_k - m_k^i}{\sigma_k^i} \right]^2 \right]. \quad (3)$$

where $m_k^i \in [m_{k1}^i, m_{k2}^i]$ is the uncertain mean, and σ_k^i is the standard deviation, with $k = 1, 2, 3, 4$ (the number of antecedents) and $i = 1, 2, \dots, 81$ (the number of M rules). Table I shows the values of the three FSs for each antecedent.

Fig. 2 shows the initial FSs of each antecedent, being the same for all inputs.

Table 1. Intervals of uncertainty for all inputs.

	m_{k1}	m_{k2}	σ_k^i
1	7.58	7.78	1.04
2	9.68	9.88	1.04
3	11.68	11.88	1.04

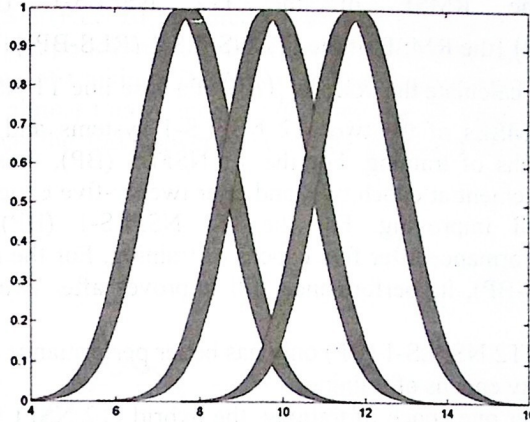


Fig. 2. Thee FSs for each of the four antecedents.

2.5. Consequent Membership Functions

The primary membership function for each consequent is a Gaussian with uncertain means, as defined in (3). Because the centre-of-sets type-reducer replaces each consequent set \tilde{G}^i by its centroid, then y_l^i and y_r^i (the M left-points and right points of interval consequent centroids) are the consequent parameters.

The initial values of the centroid parameters y_l^i and y_r^i are such that the corresponding MFs uniformly cover the output space, from 4.0 to 16.0.

3 Modeling Results

One T1 NSFLS and two IT2 NSFLS-1 FLSs were trained and used to predict the daily MXNUSD. The BP training method was used to train both the base line T1 NSFLS and IT2 NSFLS-1 forecasters, while the hybrid RLS-BP method was used to train the hybrid IT2 NSFLS-1 forecaster. For each of the two training methods, BP and RLS-BP, we ran 25 epoch computations; 1138 parameters were tuned using 2350 input-output training noisy data pairs per epoch.

The performance evaluation for the learning methods was based on root mean-squared error (RMSE) benchmarking criteria as in [1]:

$$RMSE_{S,2} (*) = \sqrt{\frac{1}{n} \sum_{k=1}^n [Y(k) - f_{S,2}(x^{(k)})]^2} \tag{4}$$

where $Y(k)$ is the output from the 100 testing noisy data pairs, $RMSE_{S,2} (*)$ stands for $RMSE_{S,2}(BP)$ [the RMSE of the IT2 NSFLS-1 (BP)], and for $RMSE_{S,2}(RLS - BP)$ [the RMSE of the IT2 NSFLS-1 (RLS-BP)]. The same formula was used in order to calculate the $RMSE_{S,1}(BP)$ of a base line T1 NSFLS (BP).

Fig. 3, shows RMSEs of the two IT2 NSFLS-1 systems and the base line T1 NSFLS for 25 epochs of training. For the T1 NSFLS (BP), there is a substantial performance improvement at epoch two, and after twenty-five epochs of training, the performance is still improving. For the IT2 NSFLS-1 (BP), there is small improvement in performance after five epochs of training. For the case of the hybrid IT2 NSFLS-1 (RLS-BP), its performance still improves after twenty-five epoch of training.

Observe that the IT2 NSFLS-1 (BP) only has better performance than T1 NSFLS-1 during the first twenty epochs of training.

In other hand, after one epoch of training, the hybrid IT2 NSFLS-1 (RLS-BP) has better performance than both T1 NSFLS (BP) and IT2 NSFLS-1 (BP) for all the twenty-five epochs. The hybrid IT2 NSFLS-1 (RLS-BP) outperforms both the T1 NSFLS (BP) and the IT2 NSFLS-1 (BP) at each epoch. The average of the $RMSE_{S,2}(BP)$ is 92.86 percent of the average of the $RMSE_{S,1}(BP)$, representing an improvement of 7.14 percent. The $RMSE_{S,2}(RLS - BP)$ improves the $RMSE_{S,1}(BP)$ by 83.52 percent and the $RMSE_{S,2}(BP)$ by 82.25 percent.

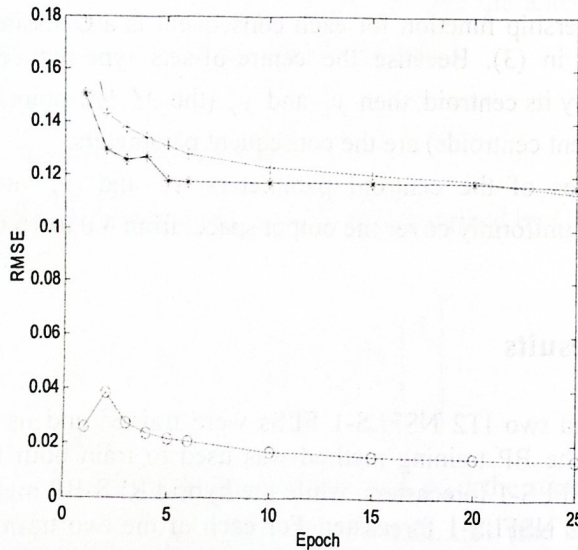


Fig. 3. T1 NSFLS (BP) (+), IT2 NSFLS-1 (BP) (*), IT2 NSFLS-1 (RLS-BP) (o).

A comparison between the predicted exchange rate after one epoch and twenty-five epochs of training using T1 NSFLS (BP), and the real price are depicted in Figures 4 and 5, respectively. Figures 6 and 7 respectively show the IT2 NSFLS-1 (BP) predictions after one epoch and twenty-five epochs of training. Observe that IT2 NSFLS-1 (BP) prediction after epoch one takes better into account the variations of the real data than its counterpart T1 NSFLS-1 (B). The predicted exchange rate using hybrid IT2 NSFLS-1 (RLS-BP) after one epoch and twenty-five epochs of training, are depicted in Figures 8 and 9, respectively. After only one epoch of training, the hybrid IT2 NSFLS-1 (RLS-BP) predictions are very close to the real values, meaning that it is feasible its application to predict and to control critical systems where there is only one chance of training. The reason is that the uncertainties presented in the training data are almost totally incorporated into the 81 rules of the IT2 NSFLS-1 forecaster, and that the fast convergence of the predicted error is improved by the RLS method.

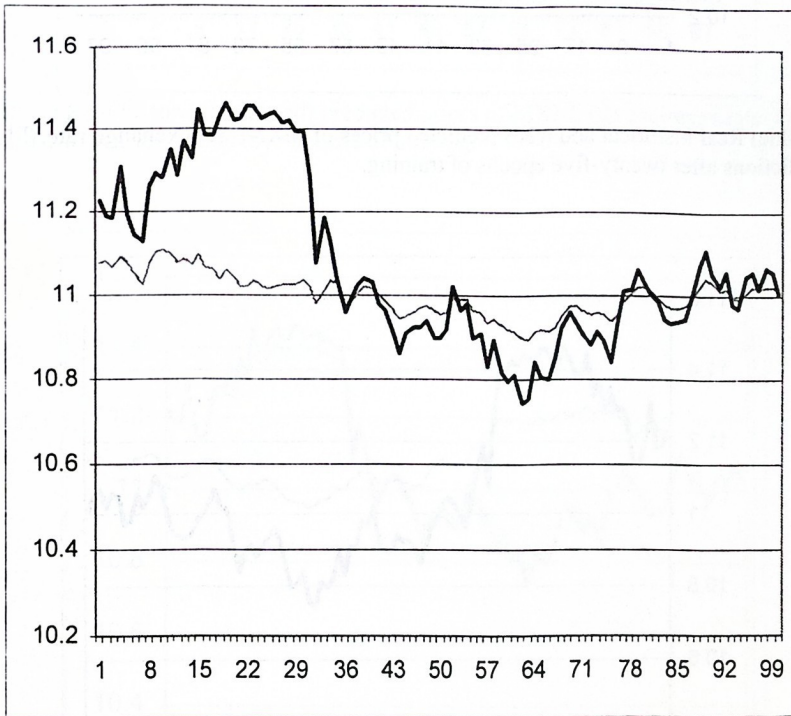


Fig. 4. (Blue) Real historical and (Red) predicted prices of MXNUSD exchange rate. T1 NSFLS (BP) predictions after one epoch of training.

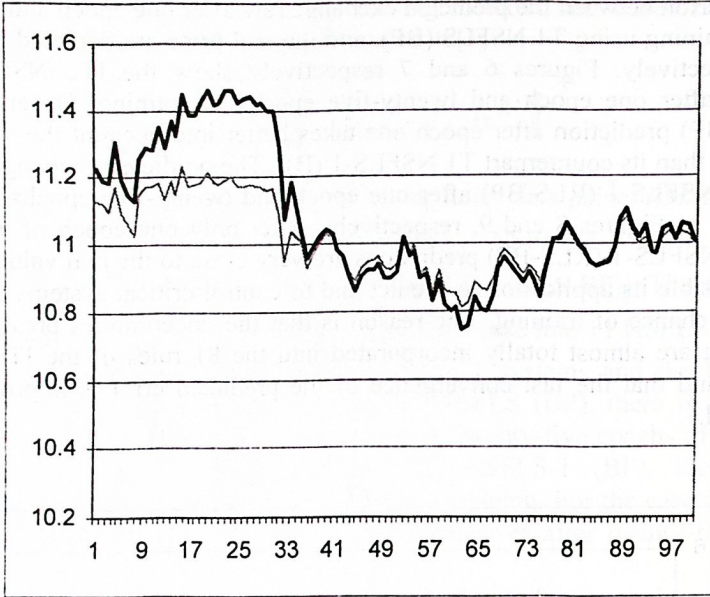


Fig. 5. (Blue) Real historical and (red) predicted prices of MXNUSD exchange rate. T1 NSFLS (BP) predictions after twenty-five epochs of training.

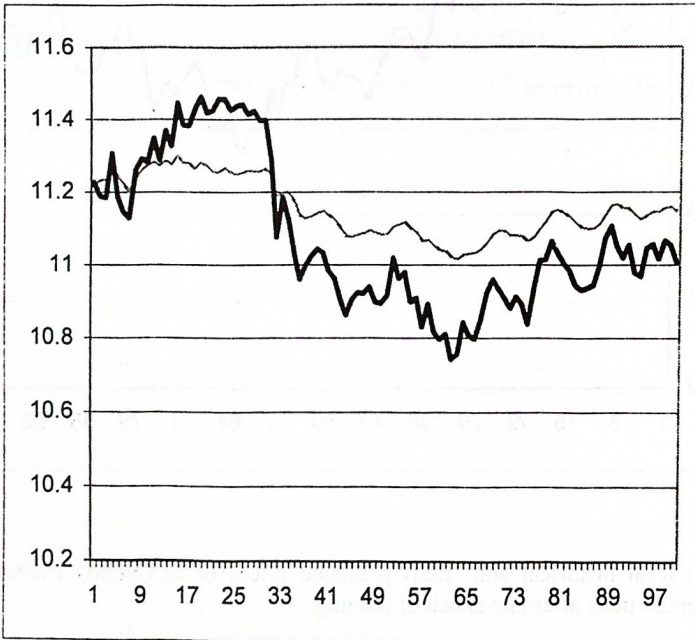


Fig. 6. (Blue) Real historical and (red) predicted prices of MXNUSD exchange rate. IT2 NSFLS-I (BP) predictions after one epoch of training.

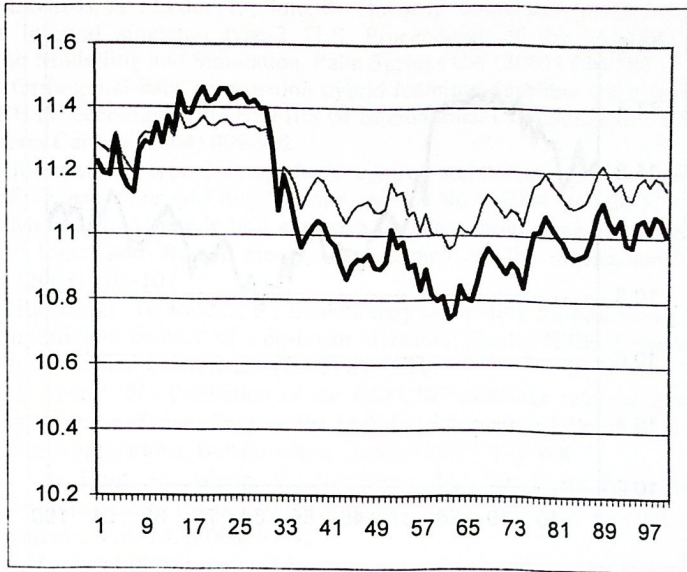


Fig. 7. (Blue) Real historical and (red) predicted prices of MXNUSD exchange rate. IT2 SFSL-1 (BP) predictions after twenty-five epochs of training.

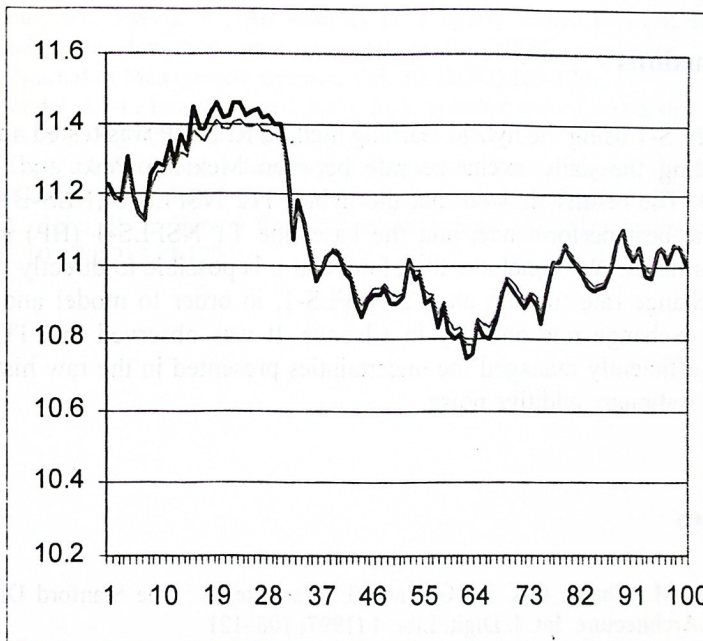


Fig. 8. (Blue) Historical and (red) predicted prices of MXNUSD exchange rate. Hybrid IT2 NSFSL-1 (RLS-BP) predictions after one epoch of training.

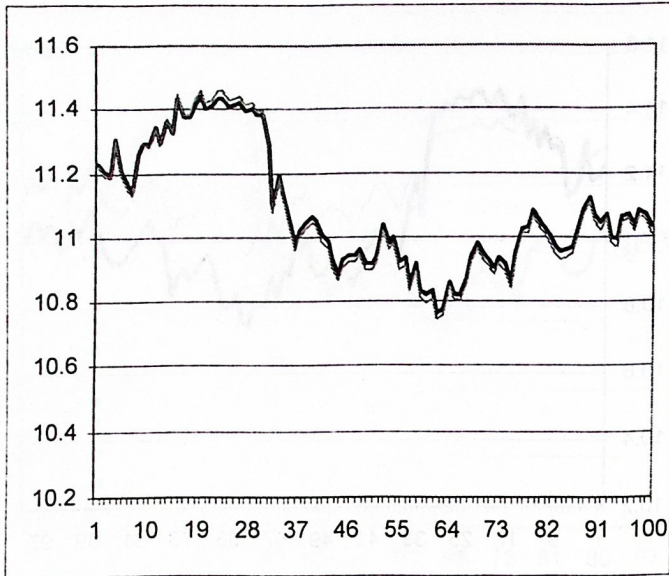


Fig. 9. (Blue) Historical and (red) predicted prices of MXNUSD exchange rate. Hybrid IT2 NSFLS-1 (RLS-BP) predictions after twenty-five epochs of training.

4 Conclusions

An IT2 NSFLS-1 using the hybrid learning method RLS-BP was tested and compared for forecasting the daily exchange rate between Mexican Peso and U.S. Dollar (MXNUSD). The results showed that the hybrid IT2 NSFLS-1 (RLS-BP) forecaster provided the best performance, and the base line T1 NSFLS-1 (BP) provided the worst performance. We conclude, therefore, that it is possible to directly use the daily data of exchange rate to train an IT2 NSFLS-1, in order to model and predict the MXNUSD exchange rate one day in advance. It was observed that IT2 NSFLS-1 forecasters efficiently managed the uncertainties presented in the raw historical data, modeled as stationary additive noise.

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